

## QuickPack® Piezo Sensors

**R. Spangler, Rev. 2, 3/4/04**

We often use QuickPack® piezo transducers as structural sensors, attaching them either directly to a data acquisition device (such as a multimeter, oscilloscope, or dsp-based analyzer), or by building a voltage follower circuit which in turn drives the data acquisition device. This Note covers the theoretical basis for this use, and provides guidance for development and data reduction. Be aware that it is also possible to get a buffered piezo sensor readout using a charge amplifier, but that option is not explicitly covered here.

Electrically, a piezo behaves as a capacitor, which can be measured using a good meter. It is also important to understand the electrical properties of the readout device, as the electrical boundary condition on the piezo impacts its sensor characteristics. In general, most readout devices have a high input impedance, as shown in Table I.

Device	Input Impedance
Fluke multimeter	>10 MΩ
Typical PC data acquisition card	500 KΩ
SigLab dynamic analyzer	1 MΩ
Tektronix 2201 oscilloscope	1 MΩ

A piezo functions as a structural sensor because of its coupling between the electrical and mechanical domains. This coupling is expressed in the sensor equation

$$E = -gT + \beta^T D \tag{1}$$

where the IEEE Standard notation has been used. That is,  $E$  is electric field,  $T$  is stress,  $D$  is electric displacement,  $\beta$  is the inverse permittivity constant matrix, and  $g$  is the matrix of piezoelectric charge coefficients. The superscript  $T$  specifies the inverse permittivity at constant (zero) stress. For piezoceramics, Equation 1 has the form

$$\begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} = - \begin{bmatrix} 0 & 0 & 0 & 0 & g_{15} & 0 \\ 0 & 0 & 0 & g_{15} & 0 & 0 \\ g_{31} & g_{31} & g_{33} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} + \begin{bmatrix} \beta_1^T \\ \beta_2^T \\ \beta_3^T \end{bmatrix} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} \quad (2)$$

where  $E_3$  is the field in the direction of the polarization vector of the ceramic. The 1, 2, and 3 stresses are extensional, again with the 3 stress in the direction of the poling, while the 4–6 stresses are shears. The  $g$  and  $\beta$  constants are related to the  $d$  and  $\epsilon$  (permittivity) constants as follows.

$$\beta^T = (\epsilon^T)^{-1} \quad g = \beta^T d \quad (3)$$

For the remainder of this Note we will consider only the QuickPack-like situation, in which we are able to assume plane stress conditions. The piezos are poled through the thickness, and electroded on the faces perpendicular to the poling. The through-thickness stress  $T_3$  is assumed to be zero. Equation (2) simplifies greatly to

$$E_3 = -g_{31}(T_1 + T_2) + \beta_3^T D_3 \quad (4)$$

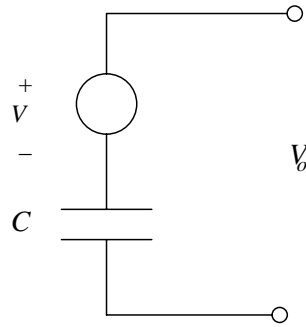
For a rectangular piezo of thickness  $t$  and area  $A$ , we simply integrate Equation (4) over the volume to get the sensor equation we can actually use.

$$\begin{aligned} E_3 A t &= -g_{31} \overline{(T_1 + T_2)} A t + \beta_3^T Q \\ \text{or} & \\ V &= -g_{31} \overline{(T_1 + T_2)} t + Q / C_p^T \end{aligned} \quad (5)$$

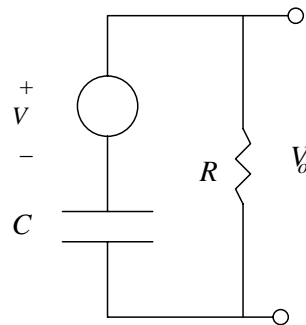
Here we have applied the definitions  $V = E_3 t$ ,  $Q = \int_A D_3 dA$  ( $Q$  is charge), and  $C_p^T = \beta_3^T t / A$  ( $C_p^T$  is the free-free piezo capacitance).

What this tells us is that the voltage from the piezo is a function of both the charge produced electrically, by the capacitor, and of the volume-averaged inplane stress produced mechanically<sup>1</sup>. The equivalent circuit to the piezo looks like this.

<sup>1</sup> A similar derivation can be produced which leaves the charge on the left hand side, and has the strain as the mechanical variable on the right hand side. Thus, if one can read charge while maintaining the voltage at zero, the piezo acts as a sensor of the average inplane strains, in both directions. This type of readout is accomplished with a charge amplifier. It cannot be accomplished by hooking the piezo directly into an instrument.



If no current flows in this circuit,  $V_o = V$ , which is proportional to the stress. This is the basis for the voltage follower. If, on the other hand, there is no stress, the piezo looks purely like a capacitor. Practically, however, some current will always flow, as the instrumentation resistance is never infinite, and there will always be some stress in the piezo (or you have designed a lousy sensor). If the input impedance of the instrument is  $R$ , then the circuit above looks like

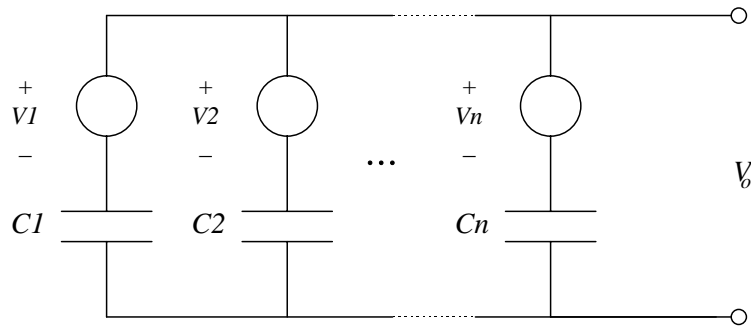


and the relation between  $V_o$  and  $V$  is, in the Laplace domain,

$$\frac{V_o}{V} = \frac{sRC}{1 + sRC} \quad (6)$$

Well above the RC corner frequency  $f_c = 1/(2\pi RC)$  this expression is approximately 1, and the voltage on the readout is directly proportional to stress. It is important to compute this frequency using the capacitance of the sensor in question and the resistance of the equipment used. If  $f_c$  is not well below the bandwidth of interest (at least 10 $\times$ ), then a high-impedance buffer circuit must be built.

If a number of piezo sensors are wired together, electrically in parallel, the equivalent circuit looks like



The Thévenin equivalent of this circuit has a single branch, with a single voltage source and a single capacitor. The total voltage and capacitance are

$$V_{tot} = \sum_{i=1}^n \frac{C_i}{C_{tot}} V_i \quad (7)$$

$$C_{tot} = \sum_{i=1}^n C_i$$

That is, the capacitance of the array is the sum of the individual piezo capacitances, and the sensor voltage is the average of all the individual sensor voltages, weighted by the capacitances. If all of the piezos are the same thickness, then the voltage is just the area-weighted average of the individual voltages, or the area-weighted average extensional stress in the piezos.